1. Mechanics

1. Force

A force is any interaction that, when unopposed, will change the motion of an object. A force can cause an object with mass to change its velocity (which includes to begin moving from a state of rest), i.e., to accelerate. Force can also be described intuitively as a push or a pull. A force has both magnitude and direction, making it a vector quantity, and it is expressed as the rate of change of momentum ($\Delta\vec{p}$) induced by the force:

$$\vec{F} = \frac{\Delta\vec{p}}{\Delta t} = \frac{\Delta\left(m\,\vec{v}\right)}{\Delta t} \tag{1}$$

It is measured in the SI unit of **Newton** (N=kg· m/s^2) and represented by the symbol ***F***.

2. Linear momentum

The linear momentum $\vec{\boldsymbol{p}}$ of an object of mass *m* moving with velocity $\vec{\boldsymbol{v}}$ is the product of its mass and velocity:

$$\vec{p} = m\vec{v} \tag{2}$$

Linear momentum is a vector quantity that is always directed in the direction of velocity. Momentum carries units of kg· m/s.

3. Velocity

In everyday usage the terms speed and velocity are interchangeable. In physics, however, there is a clear distinction between them: Speed is a scalar quantity, having only magnitude, whereas velocity is a vector, having both magnitude and direction. The velocity of an object is the rate of change of its position with respect to a frame of reference, and is a function of time. Velocity is equivalent to a specification of its speed and direction of motion (e.g. 60 km/h to the north). Velocity is an important concept in kinematics, the branch of classical mechanics that describes the motion of bodies. Its SI unit is meter per second (m/s).

The average velocity $\overline{\boldsymbol{v}}$ during a time interval $\Delta\boldsymbol{t}$ is the displacement $\Delta\boldsymbol{x}$ divided by $\Delta\boldsymbol{t}$:

$$\overline{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} \tag{3}$$

where indices *f* and *i* designate the final and initial position (*x*) and time (*t*), respectively. The instantaneous velocity $\boldsymbol{v}$ is a “special case” of the average velocity defined above if the velocity is calculated for a very short (“infinitesimally small”) time duration.

4. Angular speed

The average angular speed ω_{av} of a rotating rigid object during the time interval Δt is the angular displacement $\Delta\theta$ divided by Δt:

$$\omega_{av} = \frac{\Delta\theta}{\Delta t} \tag{4}$$

SI unit: radian per second (rad/s)

An object’s angular displacement, $\Delta\theta$, is the difference in its final and initial angles: $\Delta\theta = \theta_f - \theta_i$. It is measured in units of radians (rad).

Therefore, the angular speed of an object is the rate at which it rotates around a chosen center point: that is, the time rate of change of its angular displacement relative to the origin.

If $\Delta\theta = 2\pi$ (1 revolution), then $\Delta t = T$ (period), thus:

$$\omega = \frac{2\pi}{T} \tag{5}$$

5. Acceleration

Acceleration is the rate of change of velocity of an object with respect to time. An object's acceleration is the net result of any and all forces acting on the object, as described by Newton's Second Law. The SI unit for acceleration is meter per second squared (**m/s²**). Accelerations are vector quantities (they have magnitude and direction).

Average acceleration

An object's average acceleration over a period of time is its change in velocity $\Delta \boldsymbol{v}$ divided by the duration of the period $\Delta \boldsymbol{t}$. Mathematically,

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v_f} - \vec{v_i}}{t_f - t_i} \tag{6}$$

The numerator represents the difference between the velocity vectors $\vec{\boldsymbol{v}}_{\boldsymbol{f}}$ and $\vec{\boldsymbol{v}}_{\boldsymbol{i}}$. Indices *f* and *i* designate the final and initial quantities, respectively. These vectors may have the same magnitude, corresponding to the same speed, but if they have different directions, their difference can't equal zero. For circular motion at constant speed, the acceleration vector always points toward the center of the circle. Such an acceleration is called a **centripetal (center-seeking) acceleration**. Its magnitude is given by

$$a_c = \frac{v^2}{r} \tag{7}$$

The **tangential acceleration** of a point on a rotating object equals the distance of that point from the axis of rotation multiplied by the angular acceleration:

$$a_t = r\ \alpha \tag{8}$$

where *r* is the radius of the circular motion and α is the angular acceleration. An object's average **angular acceleration** α_{av} during the time interval $\Delta \boldsymbol{t}$ is the change in its angular speed $\Delta \boldsymbol{\omega}$ divided by $\Delta \boldsymbol{t}$:

$$\alpha_{av} = \frac{\Delta\omega}{\Delta t} = \frac{\omega_f - \omega_i}{t_f - t_i} \tag{9}$$

Instantaneous acceleration

Instantaneous acceleration, meanwhile, is the limit of the average acceleration, defined in equation (6), if Δt approaches zero.

6. Work, power

Work

In physics, a force is said to do work if, when acting, there is a displacement of the point of application in the direction of the force. Work transfers energy from one place to another, or one form to another. The work W done on an object by a constant force $\vec{\boldsymbol{F}}$ during a linear displacement along the x-axis is

$$W = F_x\ \Delta x \tag{10}$$

where F_x is the x-component of the force $\vec{\boldsymbol{F}}$ and $\Delta \boldsymbol{x} = \boldsymbol{x_f} - \boldsymbol{x_i}$ is the object's displacement. The x component of the force is defined by the following equation:

$$F_x = F\cos\theta \tag{11}$$

where F is the magnitude of the force and $\boldsymbol{\theta}$ is the angle between the direction of the force $\vec{\boldsymbol{F}}$ and the displacement $\Delta\vec{\boldsymbol{x}}$.

SI unit: **joule** (J) = newton · meter (N · m) = kg · m^2/s^2

For example, when a ball is held above the ground and then dropped, the work done on the ball as it falls is equal to the weight of the ball (a force) multiplied by the distance to the ground (a displacement).

Power

Power is the rate of doing work, the amount of energy transferred per unit time. Having no direction, it is a scalar quantity. In the International System of Units, the unit of power is the joule per second (J/s), known as the **watt**. Another common and traditional measure is horsepower. Being the rate of work, the equation for power can be written:

$$P = \frac{W}{\Delta t} = \frac{F\ \Delta x}{\Delta t} = F\ v \tag{12}$$

7. Kinetic energy

The kinetic energy KE of an object of mass *m* moving with a speed *v* is

$$KE = \frac{1}{2} m\ v^2 \tag{13}$$

SI unit: **joule** (J) = kg · m^2/s^2

The kinetic energy of an object is the energy that it possesses due to its motion.

8. Work-energy theorem

Like work, kinetic energy is a scalar quantity. Using this, we arrive at an important result known as the work–energy theorem:

The net work done on an object is equal to the change in the object's kinetic energy:

$$W_{net} = KE_f - KE_i = \Delta KE \tag{14}$$

where the change in the kinetic energy is due entirely to the object's change in speed (*f* – final, *i* – initial).

9. Gravitational potential energy

Gravitational energy is the potential energy a body with mass has in relation to another massive object due to gravity. It is potential energy associated with the gravitational field. Gravitational energy ($\boldsymbol{PE_g}$) is dependent on the masses of two bodies (*m* and *M*), their distance apart (*R*) and the gravitational constant (*G*).

$$PE_g = -G\frac{m\ M}{R} \tag{15}$$

In everyday cases only one body is accelerating measurably, and its acceleration is constant (for example, dropping a ball on Earth). For such scenarios the Newtonian formula can – for the potential energy of the accelerating body with respect to the stationary – be reduced to:

$$PE_g = m\ g\ h \tag{16}$$

where $\boldsymbol{PE_g}$ is the gravitational potential energy, *m* is the mass of the object accelerating, *g* is gravitational acceleration (*g*=9.81 m/s^2 if the object is in the gravitational field of Earth), and *h* is the distance between the bodies or vertical displacement.

10. Newton's laws

Newton's first Law

Newton's first law states that an object moves at constant velocity unless acted on by a force. The tendency for an object to maintain its original state of motion is called inertia. Mass is the physical quantity that measures the resistance of an object to changes in its velocity.

Newton's second Law

Newton's second law states that the acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass. The net force ($\sum \vec{\mathbf{F}}$) acting on an object equals the product of its mass (m) and acceleration ($\vec{\mathbf{a}}$) :

$$\sum \vec{\mathbf{F}} = m\vec{\mathbf{a}} \tag{17}$$

Solving problems with Newton's second law involves finding all the forces acting on a system and writing Equation (17) for the x-component and y-component (and z-component if applicable) separately. These equations are then solved algebraically for the unknown quantities.

Newton's third law

Newton's third law states that if two objects interact, the force $\overrightarrow{\mathbf{F}_{12}}$ exerted by object 1 on object 2 is equal in magnitude and opposite in direction to the force $\overrightarrow{\mathbf{F}_{21}}$ exerted by object 2 on object 1:

$$\overrightarrow{\mathbf{F}_{12}} = -\overrightarrow{\mathbf{F}_{21}} \tag{18}$$

An isolated force can never occur in nature.

11. Conservation laws (momentum, energy, angular momentum)

Conservation of momentum

When no net external force acts on an isolated system, the total momentum of the system ($\vec{p}$) remains constant in time. The total momentum is the vector sum of the momentum of each object:

$$\vec{p} = \sum_{i=1}^{n} \overrightarrow{p_i} \tag{19}$$

In particular, if the isolated system consists of two objects, with masses m_1 and m_2, undergoing a collision, the total momentum of the system is the same before and after the collision. Conservation of momentum can be written mathematically for this case as:

$$m_1\overrightarrow{v_{1i}} + m_2\overrightarrow{v_{2i}} = m_1\overrightarrow{v_{1f}} + m_2\overrightarrow{v_{2f}} \tag{20}$$

(i – initial, f – final).

Conservation of energy

The principle of the conservation of energy states that energy can't be created or destroyed. It can be transformed, but the total energy content of any isolated system is always constant. The same is true for the universe at large.

Conservation of angular momentum

If the net external torque acting on a system is zero, the total angular momentum of the system is constant and is said to be conserved.

$$\Delta L = \sum_{i=1}^{n} \Delta L_i = 0 \tag{21}$$

or

$$L_i = L_f \Rightarrow I_i \omega_i = I_f \omega_f \tag{22}$$

12. Angular momentum

The angular momentum (L) of a rotating object is the product of its moment of inertia (*I*) and its angular speed (ω):

$$L = I\ \omega \tag{23}$$

It carries unit of kg m^2/s.

13. Moment of inertia

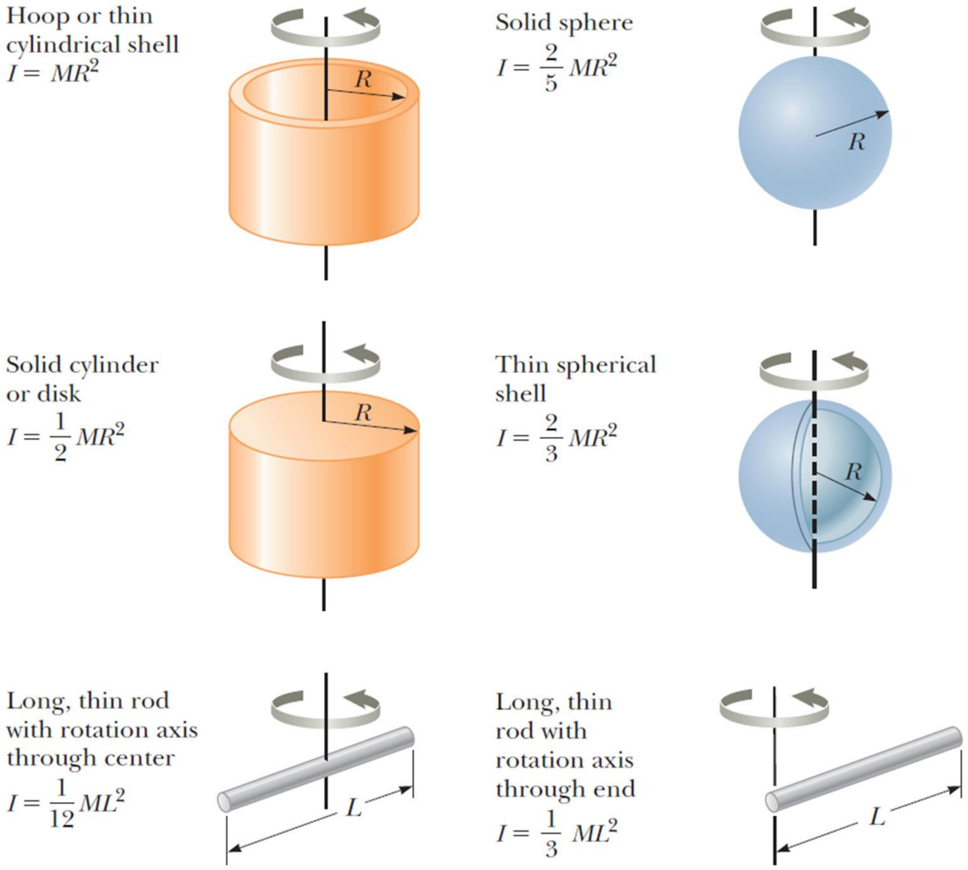
The moment of inertia of an object is its resistance to rotational acceleration about a given axis. It is always specified with respect to that axis and is defined as the sum of the products obtained by multiplying the mass of each particle of matter in a given body by the square of its distance from the axis.

The moment of inertia of an object made up of *n* point particles about an axis is given by:

$$I = \sum_{i=1}^{n} m_i r_i^2 \tag{24}$$

where m_i is the mass of one of the point particles making up the object and r_i is the distance from that point particle to the axis.

The moment of inertia of a system depends on how the mass is distributed and on the location of the axis of rotation.



Moments of Inertia for Various Rigid Objects of Uniform Composition

14. Torque

Let $\vec{F}$ be a force acting on an object, and let $\vec{r}$ be a position vector from a chosen point O to the point of application of the force. Then the magnitude of the torque $\vec{\tau}$ of the force $\vec{F}$ is given by

$$\tau = F\ r\ \sin\theta \qquad (25)$$

where r is the length of the position vector, F is the magnitude of the force, and θ is the angle between $\vec{F}$ and $\vec{r}$.

The quantity $\boldsymbol{d} = \boldsymbol{r} \sin \boldsymbol{\theta}$ is called the lever arm which is the perpendicular distance from the axis of rotation to a line drawn along the direction of the force.

By convention, counterclockwise is taken to be the positive direction, clockwise the negative direction. When an applied force causes an object to rotate counterclockwise, the torque on the object is positive. When the force causes the object to rotate clockwise, the torque on the object is negative.

The net torque acting on an object is equal to the time rate of change of the object's angular momentum (ΔL).

$$\sum \tau = \frac{\Delta L}{\Delta t} \qquad (26)$$

It means the rate of rotation of an object doesn't change, unless the object is acted on by a net torque.

The angular acceleration of an extended rigid object (α) is proportional to the net torque acting on it.

$$\sum \tau = I\alpha \qquad (27)$$

This equation is the rotational analog of Newton's second law of motion, with torque replacing force, moment of inertia replacing mass, and angular acceleration replacing linear acceleration.

15. Pressure

If F is the magnitude of a force exerted perpendicular to a given surface of area A, then the average pressure p is the force divided by the area:

$$p = \frac{F}{A} \qquad (28)$$

SI unit: pascal (Pa = N/m^2)

When a liquid is at rest, meaning that it is not flowing, we can determine its pressure at a given depth (or liquid as it fills a column) known as hydrostatic pressure:

$$p = \rho\ g\ h \qquad (29)$$

16. Hooke's law, spring force, spring constant

The force exerted by the spring, F_s, is proportional to the displacement x, or

$$F_s = -k\ x \qquad (30)$$

where k is a constant of proportionality, called spring constant, carrying units of newtons per meter. This equation is called Hooke's law, after Sir Robert Hooke, who discovered the relationship.

The value of k is a measure of the stiffness of the spring. Stiff springs have large k values, and soft springs have small k values.

The spring force is often called a restoring force because the spring always exerts a force in a direction opposite the displacement of its end, tending to restore whatever is attached to the spring to its original position.

2. Electricity and magnetism

1. Potential energy of a charged object in an electrostatic field

The electric field

An electrostatic field exerts an electric force on any charged object within the field. The electric field E at the location of a small „test" charge q_0 is defined as the electric force F_E exerted on q_0 divided by the test charge q_0:

$$E = \frac{F_E}{q_0} \qquad (31)$$

The SI unit of the electric field is newton per coulomb (N/C). When a positive test charge is used, the electric field always has the same direction as the electric force on the test charge.

Once the electric field due to a given arrangement of charges is known at some point, the force on any particle with charge q_0 placed at that point can be calculated from a rearrangement of equation (31):

$$F_E = q_0\ E \qquad (32)$$

The electric potential energy

The electric potential energy of a system of point charges is defined as the work required assembling this system of charges by bringing them close together, as in the system from an infinite distance. Consistently, the electrostatic potential energy of a point charge at a position is defined as the negative of the work done by the electrostatic force to bring it from a reference position (usually taken at infinite distance) to the given position.

The change in the electric potential energy, ΔPE, of a system consisting of an object of charge q_0 moving through a displacement Δx in a constant electric field E_x is given by

$$\Delta PE = -q_0\, E_x\, \Delta x \tag{33}$$

where E_x is the x-component of the electric field and $\Delta x = x_f - x_i$ is the displacement of charge along the x axis.

The electric potential energy is a scalar quantity with the SI unit of joule (J).

2. Electric current

The electric current is a flow of electric charge carriers. In electric circuits charges are often carried by moving electrons in a wire but can also be carried by ions in an electrolyte, or by both ions and electrons such as in an ionized gas (plasma). In metals, one or more electrons from each atom are loosely bound to the atom, and can move freely about within the metal and function as charge carriers in metal conductors. A flow of positive charges gives the same electric current, and has the same effect in a circuit, as an equal flow of negative charges in the opposite direction. Since current can be the flow of either positive or negative charges, or both, a convention is needed for the direction of current that is independent of the type of charge carriers. The direction of conventional current is arbitrarily defined as the same direction as positive charges flow.

The electric current is the rate at which charge flows through this surface. Suppose Δq is the amount of charge that flows through an area A in a time interval Δt and that the direction of flow is perpendicular to the area. Then, the current I is equal to the amount of charge divided by the time interval:

$$I = \frac{\Delta q}{\Delta t} \tag{34}$$

The SI unit of electric current is coulomb/second (C/s) or ampere (A).

In a circuit, positive charges move from regions of high potential to regions of low potential.

Macroscopic currents can be related to the motion of the microscopic charge carriers making up the current. The current in a conductor is related to the motion of the charge carriers by

$$I = n\, q\, v_d\, A \tag{35}$$

where n is the number of mobile charge carriers per unit volume, q is the charge on each carrier, v_d is the drift speed of the charges, and A is the cross-sectional area of the conductor.

3. Electric potential difference, resistance

Electric potential difference

Electric potential is closely related to electric potential energy, since it is simply the electric potential energy per unit charge. The electric potential difference *ΔV* between points *A* and *B* is the change in electric potential energy as a charge *q* moves from *A* to *B* divided by the charge *q*:

$$\Delta V = V_A - V_B = \frac{\Delta PE}{q} \tag{36}$$

The SI unit of electric potential is joule per coulomb or volt (J/C or V). Because electric potential energy is a scalar quantity, electric potential is also a scalar quantity.

Alternately, the electric potential difference is the work per unit charge that would have to be done by some force to move a charge from point *A* to point *B* in the electric field. Thus, in the process of moving through a potential difference of 1 V, the 1 C charge gains 1 J of energy.

For the special case of a uniform electric field such as that between charged parallel plates, dividing equation (33) by *q* and combining it with equation (36) gives

$$\frac{\Delta PE}{q} = \Delta V = -E\ \Delta x \tag{37}$$

Equation (37) shows that potential difference also has units of electric field times distance. It then follows that the SI unit of the electric field, the newton per coulomb, can also be expressed as volts per meter: $1N/C = 1\ V/m$.

When the zero point of electric potential is taken to be an infinite distance from the charge, the electric potential created by a point charge *q* at any distance *r* from the charge is given by

$$V = k_e \frac{q}{r} \tag{38}$$

where k_e is Coulomb's constant (k_e=8.99·10^9 N·m^2·C^{-2}).

The electric potential of two or more charges is obtained by applying the superposition principle: the total electric potential at some point due to several point charges is the algebraic sum of the electric potentials due to the individual charges.

All points on the surface of a charged conductor in electrostatic equilibrium are at the same potential (equipotential surface) and the electric potential is constant everywhere inside a conductor and equal to that same value at the surface.

Resistance

Resistance is defined as the ratio of the *ΔV* voltage across the conductor to the current *I* it carries:

$$R = \frac{\Delta V}{I} \tag{39}$$

Resistance has SI units of volts per ampere, called ohms (Ω).

For many materials, including most metals, experiments show that the resistance remains constant over a wide range of applied voltages or currents. This statement is known as Ohm's law, which is given by

$$\Delta V = I\ R \tag{40}$$

Ohm's law is an empirical relationship valid only for certain materials. Materials that obey Ohm's law, and hence have a constant resistance over a wide range of voltages, are said to be ohmic. Materials having resistance that changes with voltage or current are nonohmic. Ohmic materials have a linear current–voltage relationship over a large range of applied voltages. Nonohmic materials have a nonlinear current–voltage relationship.

4. Work done by an electric field

Since the Coulomb force is conservative and the work done by a conservative force on an object depends only on the initial and final positions of the object and not on the path taken between these two points, the ΔPE change in the electric potential energy equals to the negative of the work done by the electrostatic force:

$$\Delta PE = PE_f - PE_i = -W_{EF} \tag{41}$$

where PE_f and PE_i are the final and initial electric potential energies, respectively, and W_{EF} is the work done by the electrostatic field on the object.

Explanation of why *ΔPE* equals the negative of the work done by the field

If the potential energy of an object increases ($PE_f > PE_i$), the field performs negative work on the object while it moves from its initial position to its final position. The work is negative because the force and displacement are opposite to each other (cos 180° = –1), i.e. the force exerted by the field opposes the movement of the body.

When a small positive charge placed at point A in a uniform electric field E moves to point B (as between two equally and oppositely charged parallel plates), the W_{AB} work done on the charge by the electric field is equal to the part of the electric force F_x acting parallel to the displacement times the displacement $\Delta x = x_f - x_i$:

$$W_{AB} = F_X \ \Delta x = F_X \left(x_F - x_I\right) \tag{42}$$

When combining equations (42) and (32),

$$W_{AB} = q_0 \ E_X \left(x_F - x_I\right) \tag{43}$$

where q_0 is the charge and E_x is the vector component of E in the x-direction (not the magnitude of E). Unlike the magnitude of E, the component E_x can be positive or negative, depending on the direction of E. Finally, note that the displacement Δx, like q_0 and E_x, can also be either positive or negative, depending on the direction of the displacement.

5. Electrostatic force, Coulomb's law

Electrostatic force

An electrostatic force exists between two stationary charged particles, which has the following properties:

1. It is directed along a line joining the two particles and is inversely proportional to the square of the separation distance r, between them.
2. It is proportional to the product of the magnitudes of the charges $|q_1|$ and $|q_2|$, of the two particles.
3. It is attractive if the charges are of opposite sign and repulsive if the charges have the same sign.

Coulomb's law

Based on the previous observations, the magnitude of the electrostatic force F between two stationary charges q_1 and q_2 separated by a distance r is given by

$$F = k_e \frac{|q_1| \ |q_2|}{r^2} \tag{44}$$

where k_e is a constant called the Coulomb constant that has the value of $k_e = 8.99 \times 10^9 \ N \times m^2/C^2$ in SI units.

Equation (44), also known as Coulomb's law, applies exactly only to point charges and to spherical distribution of charges, in which case r is the distance between the two centers of charge. Like other forces, electrostatic forces obey Newton's third law; hence the forces F_{12} and F_{21} are equal in magnitude but opposite in direction.

When a number of separate charges act on the charge of interest, each exerts an electrostatic force. These electric force can all be computed separately, one at a time, then added as vectors according to the superposition principle.

6. Magnetic dipole, magnetic field, magnetic induction

Magnetic dipole

A magnet is a material or object that produces a magnetic field. One end of the magnet is called the north pole and the other the south pole (these names come from the behavior of a magnet in the presence of Earth's magnetic field) that cannot exist in isolation of each other („magnetic dipole"). Magnetic poles exert attractive or repulsive forces on each other similar to the electrical forces between charged objects, as like poles repel each other and unlike poles attract each other.

The B magnetic field at a given point in space is a vector quantity and it is specified by two properties: (1) Its direction, in which the north pole of a compass needle points at that location. (2) Its magnitude (also called strength), which is proportional to how strongly the compass needle orients along that direction. In SI units, the strength of the magnetic field is given in tesla (T).

A magnet's magnetic moment (also called magnetic dipole moment and usually denoted μ) is a vector that characterizes the magnet's overall magnetic properties. For a bar magnet, the direction of the magnetic moment points from the magnet's south pole to its north pole and the magnitude relates to how strong and how far apart these poles are. In SI units, the magnetic moment is specified in terms of Am^2 (amperes times meters squared).

Magnetic field

A magnet both produces its own magnetic field and responds to magnetic fields. The strength of the magnetic field it produces is at any given point proportional to the magnitude of its magnetic moment. The magnitude of the magnetic field is defined as

$$B = \frac{F}{q\ v\ \sin\theta} \tag{45}$$

where v is the velocity of a moving test charge q in the magnetic field B and θ is the angle between the direction of v and the direction of B. The SI unit of magnetic field is tesla (T).

In addition, when the magnet is put into an external magnetic field, produced by a different source, it is subject to a torque tending to orient the magnetic moment parallel to the field. The amount of this torque is proportional both to the magnetic moment and the external field.

The magnetic flux and the electromagnetic induction

An induced electromotive force (emf) and resulting induced current can be produced in a circuit by a changing magnetic field. The physical quantity associated with magnetism that creates an electric field is a changing magnetic flux.

The magnetic flux Φ_B through a loop of wire with area A is defined by

$$\Phi_B = B_\perp A = B\ A\ \cos\theta \tag{46}$$

where $B_\perp$ is the component of a uniform magnetic field B perpendicular to the plane of the loop and θ is the angle between B and the normal (perpendicular) to the plane of the loop. The SI unit of the magnetic flux is weber (Wb).

According to Faraday's law of induction, if a circuit contains N tightly wound loops and the magnetic flux through each loop changes by the amount $\Delta\Phi_B$ during the interval Δt, the average emf induced in the circuit during time Δt is

$$\varepsilon = -N\frac{\Delta\Phi_B}{\Delta t} \tag{47}$$

According to Lenz's law, the current caused by the induced emf travels in the direction that creates a magnetic field with flux opposing the change in the original flux through the circuit. The direction of the induced current can be determined by right-hand rule number 2: Point your right thumb in the direction that will cause the fingers on your right hand to curl in the direction of the induced field B_{ind}. In this case, the direction of the current is determined by your thumb.

An induced emf can also be produced when a conductor moves through a constant magnetic field. If a conducting bar of length L moves through a magnetic field perpendicular to the bar with a speed v in a magnetic field B, the emf induced in the bar, often called a motional emf, is

$$|\varepsilon| = B\ L\ v \tag{48}$$

7. Electric and magnetic Lorentz forces

The Lorentz force is the combination of electric and magnetic forces on a point charge due to electromagnetic fields. As described previously, the magnitude of the electric component of the Lorentz force can be determined according to equation (32).

When an object with charge q is moving with a velocity v through a magnetic field B, a magnetic force acts on it and the magnitude of that magnetic force is determined by

$$F = q\ v\ B\ \sin\theta \tag{49}$$

where θ is the angle between the direction of v and the direction of B. The direction of the magnetic force can be employed by right hand rule number 1:

1. Point the fingers of your right hand in the direction of the velocity v.
2. Curl the fingers in the direction of the magnetic field B, moving through the smallest angle.
3. Your thumb is now pointing in the direction of the magnetic force F exerted on a positive charge.

A positively charged particle will be accelerated in the same linear orientation as the E electric field, but will curve perpendicularly to both the instantaneous velocity vector v and the B magnetic field according to the right-hand rule number 1.

If a straight conductor of length L carries current I, the magnetic force F on that conductor when it is placed in a uniform external magnetic field B is

$$F = B\ I\ L\sin\theta \tag{50}$$

where θ is the angle between the direction of the current and the direction of the magnetic field. Right-hand rule number 1 also gives the direction of the magnetic force on the conductor. In this case, however, you must point your fingers in the direction of the current rather than in the direction of v.

A torque τ is exerted on a coil with N loops carrying current I in a magnetic field B and the magnitude of this torque is

$$\tau = N\ B\ I\ A\ \sin\theta \tag{51}$$

where *A* is the cross-sectional area of the loop. The magnitude of the magnetic moment of a current-carrying coil is defined by $\mu = IAN$, where *N* is the number of loops. The magnetic moment is considered a vector, μ, that is perpendicular to the plane of the loop. The angle between *B* and μ is θ.

8. Capacitance

A capacitor is a device that typically consists of two parallel metal plates separated by a distance *d*. When its plates are connected to the terminals of a voltage source, electrons are pulled off one of the plates, leaving it with a charge of *+Q*, and are transferred through the battery to the other plate, leaving it with a charge of *–Q*. The transfer of charge stops when the potential difference across the plates equals the potential difference of the battery. A capacitor is a device that stores charges and energy that can be reclaimed when needed for a specific application.

The capacitance *C* of a capacitor is the ratio of the magnitude of the charge *Q* on either conductor (plate) to the magnitude of the potential difference *ΔV* between the conductors (plates):

$$C = \frac{Q}{\Delta V} \tag{52}$$

The SI unit of capacitance is farad (F) = coulomb per volt (C/V).

The capacitance of a device depends on the geometric arrangement of the conductors. The capacitance of a typical parallel-plate capacitor with plates separated by air can be easily calculated from

$$C = \varepsilon_0 \frac{A}{d} \tag{53}$$

where *A* is the area of one of the plates, *d* is the distance between the plates, and ϵ_0 is the permittivity of free space.

3. Mechanical and electromagnetic waves

1. Wavelength, frequency

A wave is a disturbance that transfers energy through matter or space, with little or no associated mass transport. Waves consist of oscillations or vibrations of a physical medium or a field, around relatively fixed locations.

The **wavelength** is the spatial period of a periodic wave — the distance over which the wave's shape repeats. Wavelength is usually determined by considering the distance between consecutive corresponding points of the same phase, such as crests, troughs, or zero crossings. Wavelength is commonly designated by the Greek letter lambda (λ).

The time it takes for a point of the wave to complete one full cycle is known as the **period**.

The **frequency** **(f)** is the reciprocal of the period. It describes the number of cycles completed during 1 sec. The SI unit of frequency is **Hertz (Hz)**, which is defined as 1/s.

The wave speed is the speed at which a particular part of the wave moves through the medium.

The relationship of the speed, wavelength, and frequency of a wave is

$$v = f\ \lambda \tag{54}$$

This important general equation applies to many different types of waves, such as sound waves and electromagnetic waves. In electromagnetic waves, the speed of the wave is *c* (the speed of light), so the relationship is

$$c = f\ \lambda \tag{55}$$

2. Refraction of light, Snell's law

When a ray of light traveling through a transparent medium encounters a boundary leading into another transparent medium, part of the ray is reflected and part enters the second medium. The ray that enters the second medium is bent at the boundary and is said to be **refracted**.

The incident ray, the reflected ray, the refracted ray, and the normal at the point of incidence all lie in the same plane.

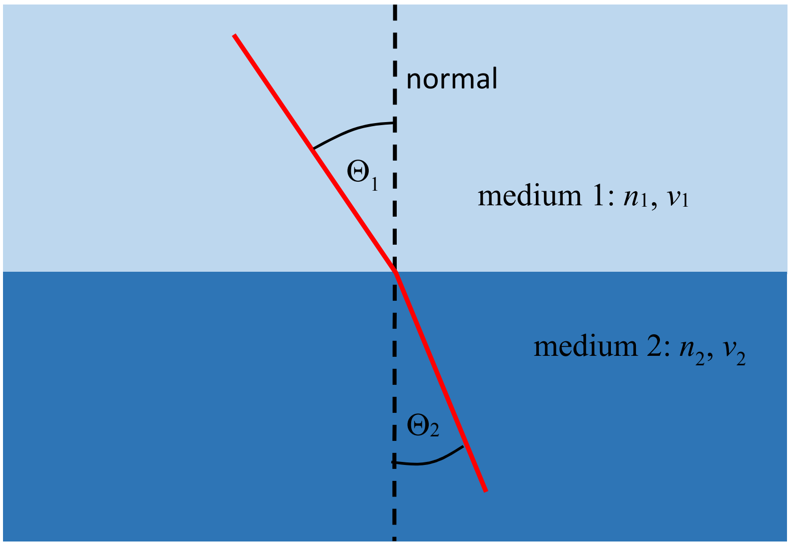
Snell's law of refraction describes the relationship between the angles of incidence (θ_1) and refraction (θ_2):

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \tag{56}$$

where n_1 and n_2 are the indices of refraction of the two media, and the angles are measured from the normal at the point of incidence.

The angle of refraction can alternatively be calculated using the speed of light in the two media:

$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{v_2}{v_1} \tag{57}$$



3. Reflection of light

When a light ray traveling in a transparent medium encounters a boundary leading into a second medium, part of the incident ray is reflected back into the first medium.

If the surface is smooth, the reflected rays are parallel to one another. This is called **specular reflection.** If the reflecting surface is rough, it reflects the rays in a variety of directions. This is known as **diffuse reflection.**

On a flat, smooth surface, if the incident and reflected rays make angles θ_1 and θ_2 with an imaginary line projected through the point of incidence, perpendicular to the surface (known as the normal):

- The incident ray, the reflected ray and the normal lie in the same plane.
- The **angle of reflection equals the angle of incidence**: $\theta_1 = \theta_2$

4. Index of refraction

When light passes from one transparent medium to another, it is refracted because the speed of light is different in the two media. The **index of refraction**, *n*, of a medium is defined as the following ratio:

$$n = \frac{\text{speed of light in vacuum}}{\text{speed of light in medium}} = \frac{c}{v} \tag{58}$$

This is a dimensionless number, greater than or equal to 1. (In vacuum: $n=1$, because $v=c$)
As light travels from one medium to another, its **frequency doesn't change**.
The equation

$$\lambda_1\, n_1 = \lambda_2\, n_2 \tag{59}$$

shows the relationship between the index of refraction and the wavelength of light.
The index of refraction of any medium can be expressed as the ratio

$$n = \frac{\text{wavelength of light in vacuum}}{\text{wavelength of light in medium}} = \frac{\lambda_0}{\lambda_n} \tag{60}$$

5. Speed of light

The speed of light in vacuum, commonly denoted *c*, is a universal physical constant. Its exact value is 299,792,458 meters per second. It is the maximum speed at which all conventional matter and hence all known forms of information in the universe can travel. When light traveling in a vacuum enters a new transparent medium, such as air, water, or glass, the speed is reduced in proportion to the index of refraction of the new material. The index of refraction is defined as the ratio of the speed of light in vacuum, *c*, and in the material, *v*:

$$n = \frac{c}{v} \tag{61}$$

6. The dual nature of light, photons, matter waves

Light has a dual nature. In some experiments it acts like a **wave**, in others like a **particle**, called a photon by Einstein.
The **energy of a photon (E)** is proportional to its frequency (f):

$$E = h\, f \tag{62}$$

where $h=6.63\times10^{-34}$ Js is Planck's constant.
According to Louis de Broglie's hypothesis, because photons have wave and particle characteristics, **all forms of matter have both properties**.
The **de Broglie wavelength** of a particle is:

$$\lambda = \frac{h}{p} = \frac{h}{m\, v} \tag{63}$$

i.e. the ratio of Planck's constant and the particle's momentum.

The frequency of matter waves (f) is related to the energy of the particle (E) by the relationship

$$f = \frac{E}{h} \tag{64}$$

7. Longitudinal and transverse waves

In a **transverse wave** the elements of the medium move in a direction **perpendicular** to the direction of the wave. Transverse waves have two parts—the crest and the trough. The crest is the highest point of the wave and the trough is the lowest. An example is a wave on a stretched string.

In a **longitudinal wave** the elements of the medium move **parallel** to the direction of the wave velocity. It consists of multiple compressions and rarefactions. An example is a sound wave in a gas.

8. Mechanical waves

A wave is a disturbance that transfers energy through matter or space, with little or no associated mass transport. A mechanical wave is a wave that is an oscillation of matter. The particles of the medium oscillate back and forth about a fixed position (with simple harmonic motion). Mechanical waves always propagate in a medium as opposed to electromagnetic waves, which do not require a medium for propagation.

The time which it takes for a particle to complete one full cycle is known as the **period**.

The **frequency** **(f)** of a wave is the reciprocal of the period. A common unit of frequency **Hertz (Hz)** is defined as 1/s.

The maximum distance the matter moves away from its equilibrium position is called the **amplitude** **(A)** of the wave.

The **wave speed (phase velocity)** is the speed at which a particular part of the wave moves through the medium.

The distance between two successive points that behave identically is called the **wavelength** **(λ)** (the Greek letter lambda).

9. Electromagnetic waves and radiation

Classically, electromagnetic radiation (EMR) consists of ***electromagnetic (EM) waves***, which are synchronized oscillations of electric and magnetic fields that propagate in each medium at the speed of light characteristic for the medium (c). Their frequency f and wavelength λ are related by the following expression:

$$c = f\,\lambda \tag{65}$$

Oscillations of electric and magnetic fields are perpendicular to each other as well as to the direction of energy and wave propagation, forming a transverse wave.

Electromagnetic waves are created by electrically charged particles undergoing acceleration.

In quantum mechanics, an alternate way of viewing EMR is that it consists of ***photons***, uncharged elementary particles with zero rest mass (*cf.* dual nature of light). Quantum effects provide additional sources of EMR, such as the transition of electrons to lower energy levels in an atom and black-body radiation. The energy of an individual photon is quantized and is greater for photons of higher frequency. This relationship is given by Planck's equation:

$$E = h\,f \tag{66}$$

where E is the energy per photon, f is the frequency of the photon, and h is Planck's constant.

Electromagnetic spectrum: position of EMR within the electromagnetic spectrum can be characterized by its frequency (or wavelength). EMRs of different frequencies are called by different names since they have different sources and effects on matter. In order of increasing frequency (or decreasing wavelength) these are: radio waves, microwaves, infrared radiation, visible light, ultraviolet radiation, X-rays and gamma rays.

10. Interference, standing waves

Interference is the superposition of waves that results in the **generation of a new wave pattern**. Interference only takes place when **coherent waves** interact with each other. Two waves are coherent if they have a constant phase difference. Interference effects can be observed with all types of waves, for example, light, radio, acoustic, surface water waves or matter waves.

Principle of wave superposition: when two or more propagating waves of same type are incident on the same point, the resultant amplitude at that point is equal to the vector sum of the amplitudes of the individual waves.

In **constructive interference** the amplitude of the resultant wave is greater than that of either of the individual waves, whereas in **destructive interference*****,*** the resultant amplitude is less than that of either individual wave. ***Maximally destructive interference*** occurs, when the phase difference is an odd multiple of π (or by other words their path difference is the half integer multiple of the wavelength), whereas for ***maximally constructive interference*** the phase difference should be an even multiple of π (or by other words their path difference is the integer multiple of the wavelength). In case of maximally constructive interference the crest of one of the waves is superimposed on the crest of the other one, whereas in maximally destructive interference the crest of one of the waves is superimposed on the trough of the other one. If the phase difference is intermediate between these two extremes, then the magnitude of the displacement of the summed waves lies between the minimum and maximum values.

Standing waves (or stationary waves): result of the superposition *(cf. interference)* of two propagating waves with the same amplitude and frequency, travelling at the same speed, but in opposite directions. The peak amplitude profile of the resultant wave does not move in space. The locations at which the amplitude is minimum are called nodes, and the locations where the amplitude is maximum are called antinodes.
E.g. stationary waves may be set up when a wave reflects back from a surface and the reflected wave interferes with the wave still travelling in the original direction.

11. Light diffraction

Light diffraction occurs when the light wave encounters an obstacle or a slit that is comparable in size to its wavelength. It is defined as the bending of light around the corners of an obstacle or aperture into the region of geometrical shadow of the obstacle. In classical physics, the diffraction phenomenon is described as the interference of waves

according to the Huygens–Fresnel principle that treats each point in the wave-front as a collection of individual spherical wavelets.
Diffraction phenomena occur not only with light waves, all types of waves, including sound waves, water waves and other kinds of electromagnetic waves, such as X-rays and radio waves. Since physical objects have wave-like properties (at the atomic level), diffraction also occurs with matter and can be studied according to the principles of quantum mechanics.

4. Atom and nuclear physics

1. Common metric prefixes

Name and symbol	Multiplier	Name and symbol	Multiplier
kilo (k)	10^3	milli (m)	10^{-3}
mega (M)	10^6	micro (μ)	10^{-6}
giga (G)	10^9	nano (n)	10^{-9}
tera (T)	10^{12}	pico (p)	10^{-12}
		femto (f)	10^{-15}
		atto (a)	10^{-18}

2. Constituent particles of atoms and their properties

The ***constituent particles of an atom*** are the electron, the proton and the neutron. However, the hydrogen-1 atom has no neutrons and the hydrogen ion has no electrons. Protons and neutrons are found in the nucleus, therefore are called nucleons. More than 99.94% of an atom's mass is in the nucleus. The protons have a positive electric charge, the electrons have a negative electric charge, and the neutrons have no electric charge. In a neutral atom the number of protons and electrons is equal.

	Mass	**Charge**
Electron	9.11×10^{-31} kg	-1.602×10^{-19} C = -1e
Proton	1.6726×10^{-27} kg ≈ 1 AU	$+1.602\times10^{-19}$ C = +1e
Neutron	1.6929×10^{-27} kg ≈ 1 AU	no charge

AU (atomic mass unit): a unit of mass used to express atomic and molecular weights, equal to one twelfth of the mass of an atom of carbon-12.
e: elementary charge, the electric charge carried by a single proton, or equivalently, the magnitude of the electric charge carried by a single electron.

Atomic radius*:* a measure of the size of the atoms, usually the mean or typical distance from the center of the nucleus to the boundary of the surrounding cloud of electrons. The value of the radius may depend on the atom's state and context. Under most definitions the radii of isolated neutral atoms range between 30 and 300 pm (trillionths of a meter), or between 0.3 and 3 ångströms (Å).

Size of the nucleus: The size of the nucleus is very small relative to the size of the atom (its diameter approx. $\times10^5$ times smaller than that of the overall atom), i.e. its radius is in the femtometer range (1–10 fm).

3. Rutherford's and Bohr's atomic models, energy levels of atomic electrons

Rutherford's planetary atomic model*:* atoms consist of a diffuse cloud of negatively charged electrons surrounding a small, dense, central core (now it is called **nucleus**), where the positive charges as well as the bulk mass of the atoms are concentrated in. Any electrons belonging to the atom were visualized as orbiting the nucleus, much as planets orbit the Sun.

Problems with the model: The laws of classical mechanics, predict that the electron will release electromagnetic radiation while orbiting a nucleus. Because the electron would lose energy, it would rapidly spiral inwards, collapsing into the nucleus, i.e. according to the model all atoms would be unstable. Also, as the electron spirals inward, the emission would rapidly increase in frequency as the orbit got smaller and faster. This would produce a continuous smear, in frequency, of electromagnetic radiation. However, atoms only emit light (that is, electromagnetic radiation) at certain discrete frequencies.

Bohr's atomic model: Niels Bohr proposed a model for the H-atom, where the electron could only have certain classical motions and can overcome the problems of Rutherford's model. The model can be generalized for H-like atoms (e.g. singly ionized helium or doubly ionized lithium) as well. Bohr's assumptions:

- The electron moves in circular orbits about the proton under the influence of the Coulomb force of attraction, which produces the electron's centripetal acceleration.
- Only certain electron orbits are stable and allowed. In these orbits no energy in the form of electromagnetic radiation is emitted, so the total energy of the atom remains constant.
- Electrons can only lose (or gain) energy by jumping from one allowed orbit to another, emitting (or absorbing) electromagnetic radiation with a frequency *f* determined by the energy difference of the levels according to the Planck relation:

  $$\Delta E = E_f - E_i = h\,f \tag{67}$$

  where h is Planck's constant, E_f and E_i are the energies of the final and initial states, respectively.
- The angular momentum of the orbiting electron is quantized:

  $$L = m_e\ v\ r = n\ \hbar,\quad n = 1,2,3... \tag{68}$$

  (m_e – mass of the electron, v – speed of the orbiting electron, $\hbar = h/2\pi$, r: radius of the orbit). The integer number n is called ***principal quantum number***, its lowest possible value is 1, which gives the smallest possible orbital radius (0.0529 nm), known as the ***Bohr-radius***. Once an electron is in this lowest orbit, it can get no closer to the proton.

Energy levels of H-atom according to the Bohr-model: Using Bohr's assumptions the energy of the allowed orbits can be calculated using the following equation:

$$E_n = \frac{-13.6}{n^2}\ (eV) \tag{69}$$

The equation above is a simplified formula, where the numerical values for all the parameters were substituted. 1 eV is the kinetic energy gained by a single electron after it accelerated across an electric potential difference of 1V (1 eV = 1.6×10^{-19} J). The lowest-energy state, or ***ground state***, corresponds to $n=1$. The other states with increasing values of *n* are called ***excited states***. When n→∞, the electron is completely removed from the atom (i.e. the atom is ionized). The minimum energy required to ionize the atom is called ***ionization energy***.

Extension of Bohr's model to heavier atoms: Bohr extended his model of hydrogen to give an approximate model for heavier atoms. Heavier atoms have more protons in the nucleus, and more electrons to cancel the charge. Each discrete orbit could only hold a certain number of electrons. After that orbit is full, the next level would have to be used. This gives the atom a ***shell structure***, with each shell corresponding to a Bohr orbit.

Although the Bohr model of the atom was shown to have many failures, the expression for the hydrogen electron energies is amazingly accurate. The Schrodinger equation for the hydrogen atom actually gave the same energies, so the Bohr model was a helpful step along the way to developing a quantum mechanical model for hydrogen.

Quantum mechanical model of the atom***:*** an atomic orbital is a mathematical function that describes the wave-like behavior of either one electron or a pair of electrons in an atom. This function can be used to calculate the probability of finding any electron of an atom in any specific region around the atom's nucleus. The term atomic orbital may also refer to the physical region or space where the electron can be calculated to be present, as defined by the particular mathematical form of the orbital.

Electron configuration of atoms: according to the quantum theory, each orbital in an atom is characterized by a unique set of values of three quantum numbers:

- **principal quantum number** (n): it characterizes the average distance of the electron from the nucleus. The electron's energy is determined mainly, but not exclusively, by the principal quantum number. It can assume values 1, 2, 3, … (small positive integers). Orbitals characterized by the same principal quantum number constitute a shell.
- **orbital (azimuthal) quantum number** (ℓ): it characterizes the angular momentum of the electron due to its motion around the nucleus, and the shape of the orbital. It can assumes values 0, 1, …, *n*-1, where *n* is the principal quantum number. Orbitals having the same principal and orbital quantum numbers constitute subshells. The simple names s orbital, p orbital, d orbital and f orbital refer to orbitals with angular momentum quantum number ℓ = 0, 1, 2 and 3 respectively.
- **magnetic quantum number** (m): it characterizes the magnetic moment of the electron associated with its orbital motion as well as the spatial orientation of the orbital. It can assume values - ℓ, - ℓ +1, …, 0, …, ℓ -1, ℓ.
- Each such orbital can be occupied by a maximum of two electrons. Besides the angular momentum due to its orbital motions, electrons have an intrinsic angular momentum called their spin. The spin of an electron is ½. An electron behaves as an elementary magnet, which can assume only two possible orientations with respect to the external magnetic field, parallel or anti-parallel. This orientation is characterized by the **spin quantum number (m_s)**, which can assume values -½ or +½.

4. Atomic number, mass number, isotopes

The **atomic number** (symbol Z) of a chemical element is the number of protons found in the nucleus of an atom. It is identical to the charge number of the nucleus. The atomic number uniquely identifies a chemical element. In an uncharged atom, the atomic number is also equal to the number of electrons.

The sum of the number of protons (atomic number, Z) and the number of neutrons (N) gives the **mass number** (A) of an atom.

Atoms with the same atomic number Z but different neutron numbers N, and hence different atomic masses, are known as **isotopes**. A little more than three-quarters of naturally occurring elements exist as a mixture of isotopes.

5. Radioactive isotopes, basic decay types

A **radionuclide** (radioactive nuclide, radioisotope or ***radioactive isotope***) is an atom that has excess nuclear energy, making it unstable. To get rid of the excess energy the radionuclide is said to undergo radioactive decay. Radionuclides occur naturally or are artificially produced in nuclear reactors, cyclotrons, particle accelerators or radionuclide generators. There are about 730 radionuclides with half-lives longer than 60 minutes. Thirty-two of those are primordial radionuclides that were created before the earth was formed.

Radioactive decay (also known as nuclear decay or radioactivity) is the process by which an unstable atomic nucleus loses energy (in terms of mass in its rest frame) by emitting radiation. The emitted radiation is high energy, ionizing radiation.

Alpha (α)-decay: the nucleus emits alpha particles consisting of two protons and two neutrons bound together into a particle identical to a helium-4 nucleus. The original atom transforms into a different one with a mass number that is reduced by four and an atomic number that is reduced by two: ${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}\alpha$.

Negative beta decay (β⁻, electron emission): an unstable atomic nucleus with an excess of neutrons may undergo β⁻ decay, where a neutron is converted into a proton, an electron, and an antineutrino ($\bar{\nu}$, the antiparticle of the neutrino): $n \rightarrow p^{+} + e^{-} + \bar{\nu}$.
The original atom transforms into a different one with an atomic number that is increased by one: ${}^{A}_{Z}X \rightarrow {}_{Z+1}{}^{A}Y + e^{-} + \bar{\nu}$. The mass number does not change.

Positive beta decay (β⁺, positron emission): Unstable atomic nuclei with an excess of protons may undergo β⁺ decay, also called positron decay, where a proton is converted into a neutron, a positron, and a neutrino (ν): $p^{+} \rightarrow n + e^{+} + \nu$. The original atom transforms into a different one with an atomic number that is reduced by one: ${}^{A}_{Z}X \rightarrow {}_{Z-1}{}^{A}Y + e^{+} + \nu$. The mass number does not change.

Gamma (γ) decay: an excited state nucleus gets rid of the excess energy by emitting high energy photons (i.e. γ-photons). It accompanies beta- or alpha-decays, when the daughter nucleus after the decay remains in excited state. Neither the atomic nor the mass number changes upon gamma-decay.

Electron capture: a process in which the proton-rich nucleus of an electrically neutral atom absorbs an inner atomic electron, usually from the K (or L) electron shell. This process thereby changes a nuclear proton to a neutron and simultaneously causes the emission of a neutrino: $p^+ + e^- \to n + \upsilon$. Following electron capture, the atomic number is reduced by one, the neutron number is increased by one, and there is no change in mass number: ${}^A_ZX \to {}_{Z-1}^{\;\;A}Y + \nu$.

5. Thermodynamics

1. Types of systems

An open system can exchange material and energy with its surroundings.
A closed system can exchange energy, but not material with its surroundings.
An isolated system cannot exchange either energy or material with its surroundings.
Any system can be converted to an isolated system by including its surroundings within the system.

2. Entropy

Entropy is a measure of disorder or randomness of a system. According to its classical thermodynamic definition if Q is the heat exchange during a reversible, constant temperature process between two equilibrium states, then the change in entropy (ΔS) between the two equilibrium states is

$$\Delta S = \frac{Q}{T} \tag{70}$$

where T is the constant temperature.
According to statistical mechanical considerations entropy (S) of a system is proportional to the thermodynamic probability (W) of the system:

$$S = k \, \ln W \tag{71}$$

where k is Boltzmann’s constant (k=1.38·10^{-23} J/K).
A microstate is a specific microscopic configuration of a thermodynamic system. In contrast, the macrostate of a system refers to its macroscopic properties, such as its temperature, pressure, volume and density. The thermodynamic probability of a system is the number of microstates leading to a given macrostate.

3. Internal energy, heat, pressure-volume work

Internal energy

The internal energy (U) is associated with the atoms and molecules of the system. The internal energy of a system is the sum of the kinetic energies of all of its particles and the energies related to the interaction between them (potential energies). The internal energy is contained within the system, excluding the kinetic and potential energies of the system as a whole due to external force fields.

Heat

The exchange of energy between two objects because of differences in their temperatures is called heat (Q). The quantity Q is positive when energy is transferred into the system by heating and negative when energy is removed from the system by cooling.

Pressure-volume work

Pressure-volume work occurs when the volume of a system changes as a result of the work. The pressure-volume work is positive when work is done on the system (for example, by compression) and it is negative when the system does positive work on its environment. The work done on a gas at constant pressure is

$$W = -p\Delta V \tag{72}$$

where p is the constant pressure, and ΔV is the change in volume. This equation shows that the work is positive when the gas is compressed (ΔV is negative).

4. First and second laws of thermodynamics

First law of thermodynamics

According to the first law of thermodynamics, when a system undergoes a change from one state to another, the **change in its internal energy** ΔU is

$$\Delta U = Q + W \tag{73}$$

where Q is the heat exchanged across the boundary between the system and the environment and W is the work done on the system.

Second law of thermodynamics

According to the second law of thermodynamics the total entropy of an isolated system remains constant if the system undergoes a reversible transition. In irreversible, spontaneous processes the total entropy of an isolated system increases. The total entropy of an isolated system can never decrease.