



Name of the course:

Course type:

Responsible lecturer:

Content:

Mathematics

compulsory

Dr. György Gát

Numerical analysis module

Features of computations by computer, error propagation. Some important matrix transformations for solving linear systems and eigenvalue problems. Gaussian elimination and its variants: its algorithms, operational complexity, pivoting. Decompositions of matrices: Schur complement, LU decomposition, LDU decomposition, Cholesky decomposition, QR decomposition. Iterative methods for solving linear and nonlinear systems: Gauss-Seidel iteration, gradient method, conjugate gradient method, Newton method, local and global convergence, quasi-Newton method, Levenberg–Marquardt algorithm, Broyden method. Solving eigenvalue problems: power method, inverse iteration, translation, QR method. Interpolation and approximation problems: Lagrange and Hermite interpolation, spline interpolation, Chebyshev-approximation. Quadrature rules: Newton-Cotes formulas, Gauss quadrature. Numerical methods for ordinary differential equations: Euler method, Runge-Kutta methods, finite-difference methods, finite element method.

Applications of ordinary differential equations module

Autonomous systems of differential equations and their phase spaces. Analytic functions of matrices, fundamental matrices of first-order, homogeneous linear differential equation systems with constant coefficients. Stability of differential equations, the theorems of Lyapunov, the direct method of Lyapunov. Boundary value problems and eigenvalue problems. Green function. Existence and uniqueness theorems. Maximum and minimum principles. Nonlinear boundary value problems. Sturm-Liouville eigenvalue problems. Rotationally symmetric elliptic problems. Diffeomorphisms and their symmetries. The application of the one-parameter symmetry group to integration of equations. Calculus of variations, the Euler–Lagrange differential equations, the invariance of the Euler–Lagrange differential equations, the canonical form of the Euler–Lagrange differential equations, the first integrals of the Euler–Lagrange differential equations. The Noether theorem. Principle of the least action.

Partial differential equations module

Examples in physics. First order equations: homogeneous linear equations, quasilinear equations and Cauchy problem for general equations. Higher order equations, the Cauchy–Kovalevskaya theorem. One-, two- and three-dimensional wave equation. Inhomogeneous wave equation. The Laplace and Poisson equation, Green functions, harmonic functions, maximum principle. Boundary value problem for the Laplace and Poisson equations. The heat equation.



Functional analysis, Hilbert spaces module

Hilbert space, the orthogonal resolution theorem, Fourier series, Bessel inequality, Gram-Schmidt orthogonalization procedure, Riesz theorem, adjoint and self-adjoint operators, projections, compact operators on Hilbert spaces, closedness of $K(H)$, spectra of compact operators, Fredholm alternative, spectral theorem of compact self-adjoint and normal operators, function calculus for compact normal operators, positive operators, Hilbert-Schmidt operators.

Completion of at least one module is required to complete the course.

Literature:

- K.E. Atkinson, Elementary Numerical Analysis. John Wiley, New York, 1993.
- K.T. Chau, Theory of Differential Equations in Engineering and Mechanics, Taylor and Francis, Hong Kong, 2018.
- V. I. Arnol'd, Ordinary differential equations, Translated from the Russian by Roger Cooke. Second printing of the 1992 edition. Universitext. Springer-Verlag, Berlin, 2006.
- V. I. Arnol'd, Mathematical methods of classical mechanics. Translated from the 1974 Russian original by K. Vogtmann and A. Weinstein. Corrected reprint of the second (1989) edition. Graduate Texts in Mathematics, 60. Springer-Verlag, New York, 1989.
- V. I. Arnold: Lectures on Partial Differential Equations, Springer, Berlin, 2004.
- J. B. Conway, A Course in Functional Analysis, Springer-Verlag, New York, 1989.